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Nonlinear propagation of ultra-low-frequency electromagnetic modes in a magnetized dusty plasma

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Abstract. A theoretical investigation has been made of nonlinear propagation of ultra-low-frequency electromagnetic waves in a magnetized two fluid (negatively charged dust and positively charged ion fluids) dusty plasma. These are modified Alfvén waves for small value of θ and are modified magnetosonic waves for large θ, where θ is the angle between the directions of the external magnetic field and the wave propagation. A nonlinear evolution equation for the wave magnetic field, which is known as Korteweg de Vries (K-dV) equation and which admits a stationary solitary wave solution, is derived by the reductive perturbation method. The effects of external magnetic field and dust characteristics on the amplitude and the width of these solitary structures are examined. The implications of these results to some space and astrophysical plasma systems, especially to planetary ring-systems, are briefly mentioned.

PACS. 52.35.Hr Electromagnetic waves (e.g., electron-cyclotron, Whistler, Bernstein, upper hybrid, lower hybrid) – 52.35.Sb Solitons; BGK modes – 52.35.Mw Nonlinear waves and nonlinear wave propagation (including parametric effects, mode coupling, ponderomotive effects, etc.)

1 Introduction

Nowadays, there has been a rapidly growing interest in understanding wave phenomena in dusty plasmas (plasmas with extremely massive and highly charged dust grains) which are ubiquitous in laboratory, space, and astrophysical plasma environments, such as, cometary tails, asteroid zones, planetary rings, interstellar medium, earth's environment, etc. [1–5]. It has been shown both theoretically and experimentally that the presence of extremely massive and highly charged static dust grains modifies the existing plasma wave spectra [6–10], whereas the dynamics of these extremely massive and highly charged dust grains introduces new eigen modes [11–17]. These modes are, for example, low frequency dust-acoustic mode [11–13], dust ion-acoustic mode [14], dust-lower hybrid mode [15], dust-drift mode [16,17], etc. A large number of investigations [11–22] have focussed attention on linear and nonlinear properties of these electrostatic modes in dusty plasmas. Recently, there has also been much interest in different new electromagnetic eigen modes in dusty plasmas and a very limited number of attempts have been made on propagation of low frequency electromagnetic modes [23–25] in such a dusty plasma system. Verheest and Buti [23] and Reddy *et al.* [24] have investigated the propagation of low frequency electromagnetic Alfvén waves (propagating along the ambient magnetic field) in a magnetized multi-species dusty plasma. Rao [25] has made a linear analysis of low frequency magnetosonic mode (propagating perpendicular to the ambient magnetic field) in a magnetized dusty plasma. The present work has considered a two fluid magnetized dusty plasma system, consisting of a highly negatively charged, extremely massive dust fluid and positively charged ion fluid, and has made an investigation of obliquely propagating electromagnetic solitary structures which are due to the modified Alfvén mode for small θ and due to the modified magnetosonic mode for large θ, where θ is the angle between the directions of the external magnetic field and the wave propagation.

The paper is organized as follows. The basic equations governing our plasma system is presented in Section 2. The Korteweg de Vries (K-dV) equation is derived by employing the reductive perturbation method in Section 3. The solitonic solution of this K-dV equation is obtained and the properties of these electromagnetic solitary structures are discussed in Section 4. Finally, a brief discussion is given in Section 5.

2 Governing equations

We consider a two-component magnetized dusty plasma system consisting of negatively charged (extremely massive) dust and positively charged ion fluids. This plasma

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system is assumed to be immersed in an external static magnetic field ($\mathbf{B}_0$) which lies in the y–z plane. Thus, at equilibrium we have $n_{i0} = Z_d n_{d0}$, where n_{i0} (n_{d0}) is the equilibrium ion (dust) number density and Z_d is the number of electrons residing on the dust grains. We assumed here that the electron number density is highly depleted due to the attachment of most of all electrons to the surface of the highly charged and extremely massive dust grains. This model is relevant to planetary ring-systems (*e.g.* Saturn's F-ring [1,14]) and to laboratory experiment [13]. The macroscopic state of this plasma system may be described by the continuity equation, the equation of motion, and the Maxwell system of equations:

$$\frac{\partial n_s}{\partial t} + \nabla\cdot(n_s\mathbf{u}_s) = 0, \tag{1}$$

$$m_s n_s(\frac{\partial}{\partial t} + \mathbf{u}_s\cdot\nabla)\mathbf{u_s} = q_s n_s(\mathbf{E} + \frac{1}{c}\mathbf{u}_s\times\mathbf{B}) - \nabla P_s, \tag{2}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \tag{3}$$

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\sum_s q_s n_s\mathbf{u}_s + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \tag{4}$$

$$\nabla\cdot\mathbf{E} = 4\pi\sum_s q_s n_s, \tag{5}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{6}$$

where s $(= \mathrm{i}, \mathrm{d})$ denotes the species, namely, ion and dust; m_s, q_s, and n_s are, respectively, mass, charge, and number density of the species s; $\mathbf{u}_s$ is the hydrodynamic velocity, $P_s = n_s k_B T_s$ with $k_B T_s$ being the thermal energy; $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field; c is the speed of light in free space.

We look at the waves propagating along the z-axis (it should be noted that the external magnetic field $\mathbf{B}_0$ makes an angle θ with the z-axis) where all wave quantities will depend only on z and t. We consider cold dust fluid, quasi-neutrality condition, and negligible the contribution due to the displacement current. Then, one can reduce our basic equations (1–6) to

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}_d) = 0, \tag{7}$$

$$\frac{\mathrm{d}\mathbf{u}_d}{\mathrm{d}t} + \left(\frac{V_A^2}{\omega_{ci}}\right)\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{n}(\nabla\times\mathbf{B}^{\mathrm{T}})\right] - V_A^2\left[\frac{1}{n}(\nabla\times\mathbf{B}^{\mathrm{T}})\times\mathbf{B}^{\mathrm{T}}\right] + C_d^2\left[\frac{1}{n}\nabla n\right] = 0, \tag{8}$$

$$\frac{\mathrm{d}\mathbf{B}^{\mathrm{T}}}{\mathrm{d}t} + \mathbf{B}^{\mathrm{T}}(\nabla\cdot\mathbf{u}_d) - (\mathbf{B}^{\mathrm{T}}\cdot\nabla)\mathbf{u}_d - \frac{1}{\omega_{cd}}\left[\nabla\times\left(\frac{\mathrm{d}\mathbf{u}_d}{\mathrm{d}t}\right)\right] = 0, \tag{9}$$

where n is the dust particle number density normalized to n_{d0}; $\mathbf{B}^{\mathrm{T}}$ is the total magnetic field vector normalized to B_0; $\mathrm{d}/\mathrm{d}t = \partial/\partial t + u_{dz}\nabla$; $V_A = B_0/\sqrt{4\pi n_{d0}(Z_d m_i + m_d)}$; $C_d = \sqrt{Z_d T_i/(Z_d m_i + m_d)}$; $\omega_{ci} = eB_0/(m_i c)$; $\omega_{cd} = Z_d e B_0/(m_d c)$.

3 Derivation of the K-dV equation

To study electromagnetic solitary waves in the dusty plasma system under consideration, we construct a weakly nonlinear theory of the low frequency electromagnetic waves. We first consider a frame of reference moving with the solitary wave and assume the amplitude to be a small quantity of order of ϵ. These assumptions lead to the scaling of the independent variables through the standard Korteweg-de Vries (K-dV) stretching [26–29]

$$\left.\begin{aligned}\xi &= \epsilon^{1/2}(z - V_p t)\\ \tau &= \epsilon^{3/2}t\end{aligned}\right\} \tag{10}$$

where V_p is the wave phase velocity and the small quantity ϵ is a measure of the solitary wave amplitude, *i.e.*, a measure of the weakness of the dispersion or of the nonlinear effects. We can expand the perturbed quantities n, $u_{dz,y,x}$, and $B^{\mathrm{T}}_{x,y}$ (it should be noted that $B^{\mathrm{T}}_z = \cos\theta$) about their equilibrium values in powers of ϵ as:

$$\left.\begin{aligned}n &= 1 + \epsilon n^{(1)} + \epsilon^2 n^{(2)} + \cdots\\ u_{dz} &= 0 + \epsilon u^{(1)}_{dz} + \epsilon^2 u^{(2)}_{dz} + \cdots\\ u_{dy} &= 0 + \epsilon u^{(1)}_{dy} + \epsilon^2 u^{(2)}_{dy} + \cdots\\ u_{dx} &= 0 + \epsilon^{3/2} u^{(1)}_{dx} + \epsilon^{5/2} u^{(2)}_{dx} + \cdots\\ B^{\mathrm{T}}_x &= 0 + \epsilon^{3/2} B^{(1)}_x + \epsilon^{5/2} B^{(2)}_x + \cdots\\ B^{\mathrm{T}}_y &= \sin\theta + \epsilon B^{(1)}_y + \epsilon^2 B^{(2)}_y + \cdots\end{aligned}\right\}. \tag{11}$$

Now, substituting (10, 11) in (7–9) and equating the various powers of ϵ, a sequence of equations can be obtained. Equating the coefficients of $\epsilon^{3/2}$ one obtains a set of equations which can be simplified as

$$\left.\begin{aligned}u^{(1)}_{dz} &= V_p(\frac{V_A^2}{V_p^2 - C_d^2})\sin\theta B^{(1)}_y\\ n^{(1)} &= (\frac{V_A^2}{V_p^2 - C_d^2})\sin\theta B^{(1)}_y\\ u^{(1)}_{dy} &= -\frac{V_A^2}{V_p}\cos\theta B^{(1)}_y\\ V_p^2 &= \frac{1}{2}[V_A^2 + C_d^2 \pm \sqrt{(V_A^2 + C_d^2)^2 - 4C_d^2 V_A^2\cos^2\theta}]\end{aligned}\right\}. \tag{12}$$

The last equation represents the general dispersion relation for the low frequency electromagnetic waves in the magnetized dusty plasma under consideration. It is obvious that for parallel propagation ($\theta = 0$) this (with + sign) reduces to the simplest form of the dispersion relation for the dust Alfvén mode [23,24], in which the magnetic pressure ($B_0^2/4\pi$) gives rise to the restoring force and the plasma mass density ($n_{i0}m_i + n_{d0}m_d$) provides the inertia, and that for perpendicular propagation ($\theta = 90°$) this reduces to the dispersion relation for the magnetosonic mode [25] in which the sum of magnetic and ion-thermal pressures ($B_0^2/4\pi + n_{i0}k_B T_i$) gives rise to the restoring force and the plasma mass density ($n_{i0}m_i + n_{d0}m_d$) provides the inertia. It should be mentioned that for zero external magnetic field ($V_A = 0$) this magnetosonic mode turns into the dust-acoustic mode studied by a number of authors in last few years [11–13,19–22].

The next set of the equations, which are obtained by substituting (10, 11) into (7–9) and equating of ϵ^2, can be written as

$$\left.\begin{aligned} u_{\mathrm{d}x}^{(1)} &= \left[\frac{1}{\omega_{\mathrm{ci}}} - \left(\frac{1}{\omega_{\mathrm{cd}}} - \frac{1}{\omega_{\mathrm{ci}}}\right)\frac{V_A^2\cos^2\theta}{V_{\mathrm{p}}^2 - V_A^2\cos^2\theta}\right] V_A^2 \frac{\partial B_y^{(1)}}{\partial\xi}, \\ B_x^{(1)} &= \left(\frac{1}{\omega_{\mathrm{cd}}} - \frac{1}{\omega_{\mathrm{ci}}}\right)\left(\frac{V_A^2 V_{\mathrm{p}}}{V_{\mathrm{p}}^2 - V_A^2\cos^2\theta}\right)\cos\theta \frac{\partial B_y^{(1)}}{\partial\xi}\cdot \end{aligned}\right\} \tag{13}$$

The last set of the equations, which are obtained by substituting (10, 11) into (7–9) and equating of $\epsilon^{5/2}$, are expressed as

$$\frac{\partial n^{(1)}}{\partial\tau} - V_p\frac{\partial n^{(2)}}{\partial\xi} + \frac{\partial}{\partial\xi}[u_{\mathrm{d}z}^{(2)} + n^{(1)}u_{\mathrm{d}z}^{(1)}] = 0, \tag{14}$$

$$\begin{aligned} &\frac{\partial u_{\mathrm{d}z}^{(1)}}{\partial\tau} - V_p\frac{\partial u_{\mathrm{d}z}^{(2)}}{\partial\xi} - [V_{\mathrm{p}}n^{(1)} - u_{\mathrm{d}z}^{(1)}]\frac{\partial u_{\mathrm{d}z}^{(1)}}{\partial\xi} \\ &+ V_A^2\sin\theta\frac{\partial B_y^{(2)}}{\partial\xi} + V_A^2 B_y^{(1)}\frac{\partial B_y^{(1)}}{\partial\xi} + C_{\mathrm{d}}^2\frac{\partial n^{(2)}}{\partial\xi} = 0, \end{aligned} \tag{15}$$

$$\begin{aligned} &\frac{\partial u_{\mathrm{d}y}^{(1)}}{\partial\tau} - V_{\mathrm{p}}\frac{\partial u_{\mathrm{d}y}^{(2)}}{\partial\xi} - V_{\mathrm{p}}n^{(1)}\frac{\partial u_{\mathrm{d}z}^{(1)}}{\partial\xi} + u_{\mathrm{d}z}^{(1)}\frac{\partial u_{\mathrm{d}y}^{(1)}}{\partial\xi} \\ &- \frac{V_A^2 V_{\mathrm{p}}}{\omega_{\mathrm{ci}}}\frac{\partial^2 B_x^{(1)}}{\partial\xi^2} - V_A^2\cos\theta\frac{\partial B_y^{(2)}}{\partial\xi} = 0, \end{aligned} \tag{16}$$

$$\begin{aligned} &\frac{\partial B_y^{(1)}}{\partial\tau} - V_{\mathrm{p}}\frac{\partial B_y^{(2)}}{\partial\xi} + u_{\mathrm{d}z}^{(1)}\frac{\partial B_y^{(1)}}{\partial\xi} + B_y^{(1)}\frac{\partial u_{\mathrm{d}z}^{(1)}}{\partial\xi} \\ &+ \sin\theta\frac{\partial u_{\mathrm{d}z}^{(2)}}{\partial\xi} - \cos\theta\frac{\partial u_{\mathrm{d}y}^{(2)}}{\partial\xi} + \frac{V_{\mathrm{p}}}{\omega_{\mathrm{cd}}}\frac{\partial^2 u_{\mathrm{d}x}^{(1)}}{\partial\xi^2} = 0. \end{aligned} \tag{17}$$

Now, using (12–17), one can eliminate $n^{(2)}$, $u_{\mathrm{d}y,z}^{(2)}$, and $B_y^{(2)}$ and can obtain [30]

$$\frac{\partial B_y^{(1)}}{\partial\tau} + AB_y^{(1)}\frac{\partial B_y^{(1)}}{\partial\xi} + D\frac{\partial^3 B_y^{(1)}}{\partial\xi^3} = 0, \tag{18}$$

where

$$\left.\begin{aligned} A &= \frac{A_1}{C} \\ D &= \frac{D_1}{C} \\ A_1 &= V_{\mathrm{p}}^3\sin\theta\left[3(V_{\mathrm{p}}^2 - C_{\mathrm{d}}^2)V_A^2 + 2C_{\mathrm{d}}^2 V_A^2\left(\frac{V_A^2}{V_{\mathrm{p}}^2 - C_{\mathrm{d}}^2}\right)\right] \\ D_1 &= \frac{V_A^2 V_{\mathrm{p}}^3}{\omega_{\mathrm{cd}}\omega_{\mathrm{ci}}}(V_{\mathrm{p}}^2 - C_{\mathrm{d}}^2)^2\left[1 - \frac{V_A^2\cos^2\theta}{V_{\mathrm{p}}^2 - V_A^2\cos^2\theta}\frac{(\omega_{\mathrm{ci}} - \omega_{\mathrm{cd}})^2}{\omega_{\mathrm{cd}}\omega_{\mathrm{ci}}}\right] \\ C &= V_A^2 V_{\mathrm{p}}^2(V_{\mathrm{p}}^2 + C_{\mathrm{d}}^2)\sin^2\theta \\ &\quad + (V_{\mathrm{p}}^2 - C_{\mathrm{d}}^2)^2(V_{\mathrm{p}}^2 + V_A^2\cos^2\theta) \end{aligned}\right\}. \tag{19}$$

This equation (18), which is nothing but the K-dV equation, describes the nonlinear propagation of low frequency electromagnetic waves (due to the oscillations of dust particles and ions) in the magnetized dusty plasma system under consideration.

4 Solitonic solution of the K-dV equation

The steady state solitonic solution of the K-dV equation is obtained by considering a moving coordinate (moving with speed u_0) $\eta = \xi - u_0\tau$, and imposing the appropriate boundary conditions, *viz.*, $B_y^{(1)} \to 0$, $\mathrm{d}B_y^{(1)}/\mathrm{d}\eta \to 0$, $\mathrm{d}^2 B_y^{(1)}/\mathrm{d}\eta^2 \to 0$ at $\eta \to \pm\infty$. Thus, one can express the steady state solitonic solution of this K-dV equation as

$$B_y^{(1)} = B_{ym}^{(1)}\mathrm{sech}^2[(\xi - u_0\tau)/\delta], \tag{20}$$

where the amplitude $B_{ym}^{(1)}$ (normalized to B_0) and the width δ are given by

$$\left.\begin{aligned} B_{ym}^{(1)} &= 3u_0/A, \\ \delta &= \sqrt{4D/u_0}. \end{aligned}\right\} \tag{21}$$

It is obvious from (19) that since $V_{\mathrm{p}} \geq C_{\mathrm{d}}$, A is always positive. Therefore, the sign of D defines the nature of these electromagnetic solitary structures. For $D < 0$, we find subsonic ($u_0 < 0$) rarefactive electromagnetic solitary structures and for $D > 0$, we find supersonic ($u_0 > 0$) compressive electromagnetic solitary structures. It is shown that for the pure magnetosonic waves ($\theta = \pi/2$) D is always positive, *i.e.*, only compressive electromagnetic solitons exist and for $\theta < \pi/2$ there exists rarefactive electromagnetic K-dV solitons if

$$\frac{V_A^2\cos^2\theta}{V_{\mathrm{p}}^2 - V_A^2\cos^2\theta} > \frac{\omega_{\mathrm{cd}}\omega_{\mathrm{ci}}}{(\omega_{\mathrm{ci}} - \omega_{\mathrm{cd}})^2}\cdot \tag{22}$$

It should be mentioned here that these K-dV solitons do not exist for the shear Alfvén mode ($\theta = 0$) since in this case $\cos\theta = 1$ and $V_{\mathrm{p}} = V_{\mathrm{A}}$. In this case one should derive the derivative nonlinear Schrödinger (DNLS) equation [31] and study the properties of these Alfvén solitons which are investigated elsewhere [32].

In order to have some numerical appreciation of these results we have made numerical calculations for usual space and astrophysical dusty plasma parameters [3,33]: $m_{\mathrm{d}} = 10^{-12}$ gm, $m_{\mathrm{i}} = 10^{-23}$ gm, $n_{\mathrm{d}0} = 10$ cm^{-3}, $T_{\mathrm{i}} = 0.1$ eV, $Z_{\mathrm{d}} = 10^5$–10^7, and $B_0 = 0.01$–1.0 Gauss, and have shown how the height and the width of these electromagnetic solitary structures change with the magnitude of the external magnetic field (B_0) and of the dust grain charge (Z_{d}). These are displayed in Figures 1 and 2. These show that as we increase the magnitude of the external magnetic field, both the amplitude and the width of these solitary structures increase. These also show that as we increase the magnitude of the dust grain charge, the amplitude decreases whereas the width increases.

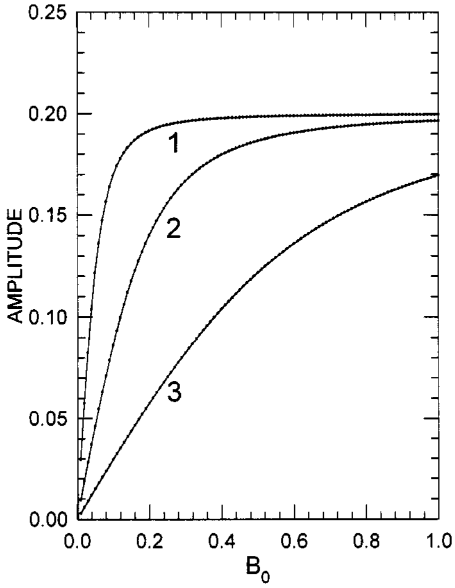

Fig. 1. Variation of the amplitude ($B_{ym}^{(1)}$) of the solitary waves with the external magnetic field (B_0) for $u_0 = C_{\rm d}$, $m_{\rm d} = 10^{-12}$, $m_{\rm i} = 10^{-23}$, $n_{\rm d0} = 10$ cm^{-3}, $T_{\rm i} = 0.1$ eV, $Z_{\rm d} = 10^5$ (curve 1), $Z_{\rm d} = 10^6$ (curve 2), and $Z_{\rm d} = 10^7$ (curve 3).

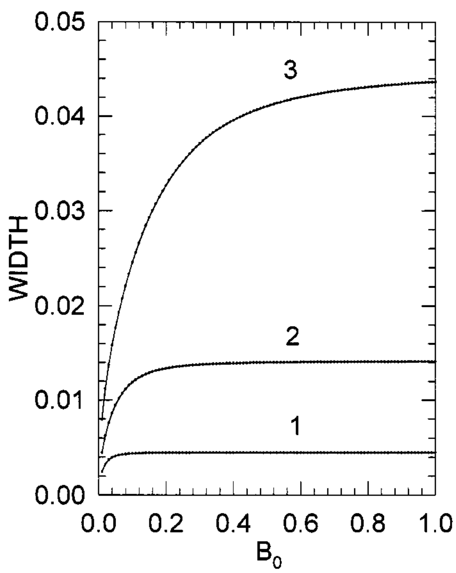

Fig. 2. Variation of the width (δ) of the solitary waves with the external magnetic field (B_0) for $u_0 = C_{\rm d}$, $m_{\rm d} = 10^{-12}$, $m_{\rm i} = 10^{-23}$, $n_{\rm d0} = 10$ cm^{-3}, $T_{\rm i} = 0.1$ eV, $Z_{\rm d} = 10^5$ (curve 1), $Z_{\rm d} = 10^6$ (curve 2), and $Z_{\rm d} = 10^7$ (curve 3).

5 Discussion

A self consistent and general description of weakly nonlinear low-frequency electromagnetic waves (propagating obliquely with the ambient magnetic field) in a magnetized two component dusty plasma consisting of highly negatively charged, extremely massive dust and positively charged ion fluids, has been presented. This investigation, where the nonlinear K-dV equation and its solitonic solution are derived, is based on the reductive perturbation method and fluid theory. It is assumed here that the electron number density is highly depleted due to the attachment of most of all electrons to the surface of highly charged and extremely massive dust grains. This assumption is relevant to planetary ring-systems (*e.g.* Saturn's F-ring [1,14]) and laboratory experiment [13]. The cold dust fluid and the quasi-neutrality condition are also considered. The results, which have been found in this investigation, may be pointed out as follows:

(i) It has been found that there exists linear/nonlinear ultra-low-frequency dust-electromagnetic modes (modified shear or compressional Alfvén waves propagating obliquely with the ambient magnetic filed) in the magnetized dusty plasma system under consideration. It is also observed that, for parallel propagation ($\theta = 0$), the linear mode becomes the shear dust-Alfvén mode which doesn't compress either the magnetic field or the plasma density, but bends (shears) the magnetic field lines and that, for finite θ, this linear mode reduces to the compressional Alfvén mode which causes compression of both the plasma density and magnetic field lines;

(ii) it has been found that the dusty plasma system may also support obliquely propagating electromagnetic solitary waves, associated with the compressional Alfvén or magnetosonic mode. It is shown here that, for perpendicular propagation ($\theta = \pi/2$), there exists only super magnetosonic ($u_0 > 0$) compressive solitary structures [30]. However, for a certain range of θ, satisfying the condition (22), super magnetosonic ($u_0 > 0$) compressive solitary structures may change to sub-magnetosonic ($u_0 < 0$) rarefactive ones. The nonlinear analysis presented here is not valid for exact parallel propagation ($\theta = 0$) in which case one should derive the derivative nonlinear Schrödinger (DNLS) equation [31] and examine the properties of these Alfvén solitons [32];

(iii) it has been shown that as we increase the magnitude of the external magnetic field, both the amplitude and the width of these solitary structures increase. It is also found here that as we increase the magnitude of the dust grain charge, the amplitude decreases whereas the width increases.

It should be mentioned here that for our numerical calculations we choose values of different parameters which are, of course, typical for a number of space dusty plasma systems, particularly, for planetary ring systems (typical approximate dusty plasma parameters [1,3,33] in planetary ring systems are $n_{\rm d0} \simeq 10^{-6}$–10 cm^{-3}, $Z_{\rm d} \simeq$ 10–10^5, $T_{\rm i} \simeq 0.01$–1.0 eV, $m_{\rm d} \simeq 10^{-13}$–$10^{-7}$ gm, $B_0 \simeq 10^{-3}$–10 G, etc.) and cometary environments (typical approximate plasma parameters [1,3,33] in dust regions (tails) of Halley's comet are $n_{\rm d0} \simeq 10^{-7}$–10 cm^{-3}, $Z_{\rm d} \simeq 10^4$–10^6, $T_{\rm i} \simeq 0.001$–0.1 eV, $m_{\rm d} \simeq 10^{-13}$–$10^{-7}$ gm, $B_0 \simeq 10^{-3}$–0.1 G, etc.)

It may be pointed out that these results might be useful for understanding the electromagnetic disturbances in some space and astrophysical dusty plasma systems, especially in planetary ring systems, because the planetary magnetic field lines from a nearly aligned dipole (Jupiter, Saturn, etc.) are perpendicular to the equatorial plane in which the bulk of the ring material moves.

It may be added here that the effects of inhomogeneity in plasma density and in the ambient magnetic field on these electromagnetic solitary structures and their instabilities are also problems of great importance but beyond the scope of the present work.

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